Let $f: B^3 \to B^6$ be the linear code with matrix

$$\begin{pmatrix} I_{3\times 3} \\ L \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

① Complete the table of values for f.

	0	1	2	3	4	5	6	7
X	000	(0,0,1)	010	011	100	101	110	111
$f(x)$	000000	001101	010011	011110	100111	101010	110100	

What is the smallest distance between two codewords in $W = \text{Image}(f)$?

② Complete the "coset decoding table" for f, using the parity matrix

$$H = \left(L \;\; I_{(6-3)\times(6-3)}\right) = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} : B^6 \to B^3$$

	0	1	2	3	4	5	6	7
000	000000	001101	010011	011110				111001
001	000001	001100	010010					
010	000010	001111	010001	011100	100101			

The steps are: To finish a row, add the "error" vector from column 0 to the various entries in row 000. To start a new row, choose a 6-bit number, not already listed, with the smallest number of bits. Write it under column 0, and label the row by the output of H when applied to this number.

(3) Since the smallest distance between codewords is 3, 1 error in the transmission of a codeword from W can always be corrected.

- 011100 is not a codeword, but if at most one bit of error has occured in this representation, the original word must have been $011110 = 3$.

- I send you a number from 0 to 7 using f, but you receive 101000 (which is not a value of f). What did I most likely originally send?

- What number in your table is in row 011 and column 0?

$e =$ ____

This vector can correct any word $u \in B^6$ such that $H(u) = 011$.

Find 3 non-codewords u such that $H(u) = 011$.

____ , ____ , ____

For these 3, what is $u + e$?

____ , ____ , ____