1. Place two identical dice on your desk. Practice the following moves:

   (i) Pick up one die without rotating it. Hold it with 2 fingers, one on a face and one on the opposite face. Rotate the die, without moving your 2 fingers, until the faces return to a place where faces were, edges return to where edges were, etc.

   (ii) Do the same thing with 2 fingers on a vertex and for opposite vertex.

   What type of move has been left off this list?

   Come up with a notation (a symbol) for each of the symmetries of the types described above as well as the third type. List them.

2. Rotate the cube backwards once, then left once, then forward once, then right once. What is the resulting motion of the die? Express your answer as an equation, i.e.,

$$a \cdot b \cdot c \cdot d = e$$

   (Hint: What happens to the numbers labelling the faces? The resulting motion is on your list.)

* 3. How many motions, or symmetries, of the cube are there? Why is your list from part 1 the full list?

4. The "inverse" of a symmetry is the opposite motion. The inverse of a symmetry g is usually denoted g^{-1} or $\bar{g}$. Since the product "$\cdot$" of two symmetries is the composition of transformations, we have

$$g \cdot g^{-1} = 1$$

(Here 1 is the "do nothing" symmetry.)

Compute the following products, and express them as equations using the symbols on your list:

(i) The rotation backward by 180°, followed by the rotation to the right by 180°

(ii) The rotation backward by 90°, followed by leftward by 90°, followed by backward by 90°.

(iii) The rotation backward by 90° twice, followed by rightward by 90° twice, followed by forward by 90° twice, followed by leftward by 90° twice.

(iv) A "backward" rotation holding 2 vertices, followed by another backward rotation holding 2 other vertices.

5. One of the symmetries on your list sends face 1 to face 2,
face 2 to face 3,
face 5 to face 4,
face 4 to face 6,
face 6 to face 5,
and face 3 to face 1.

We can call this element by the denotation:

$$(1\,2\,3)(5\,4\,6)$$

(We could also call it $(3\,1\,2)(5\,4\,6)$ or $(4\,6\,5)(2\,3\,1)$.)

Would we call any of the elements $(1\,4\,5)(2\,6\,3)$?

How about $(1\,4\,3)(2\,6\,5)$? $(1\,2\,6\,5)(3)(4)$?

Or $(1\,6)(2\,5)(3)(4)$?

$(2\,5)(1\,4)(3\,6)$?

What is the rule for deciding which notations arise?

6. If R is a rotation by x degrees and S is any symmetry, $S \cdot R \cdot S^{-1}$ is a rotation by x degrees also. Verify this for some of the rotations R and symmetries S belonging to your list of symmetries of the die.

(R and SRS^{-1} are called "conjugate".)