

4-jet of curve $(x(t), y(t))$:

$$\begin{bmatrix} N_1 \\ M_1 \\ N_2 \\ M_2 \\ N_3 \\ M_3 \\ N_4 \\ M_4 \end{bmatrix} = \begin{bmatrix} y' \\ x' \\ x'y'' - y'x'' \\ -(x'y'' - y'x'') \\ -3x''(x'y'' - x''y') + x'(x'y''' - x'''y') \\ 3y''(x'y'' - x''y') - y'(x'y''' - x'''y') \\ 15x''^2(x'y'' - y'x'') + x'^2(x'y'''' - y'x''''') - 10x'x'''(x'y'' - y'x'') - 6x'^2(x''y''' - y''x''') \\ -15y''^2(x'y'' - y'x'') - y'^2(x'y'''' - y'x''''') + 10y'y'''(x'y'' - y'x'') + 6y'^2(x''y''' - y''x''') \end{bmatrix}$$

$\in \mathbb{P}(1, 1, 3, 3, 5, 5, 7, 7)$

Satisfies:

$$\begin{aligned} 0 &= N_2 + M_2 \\ 0 &= N_1N_3 + M_1M_3 - 2N_2^2 \\ 0 &= N_1^2N_4 + M_1^2M_4 - 5N_2^3 + 5M_1N_2M_3 \end{aligned}$$

Euclidean isometry invariants:

$$\kappa = \frac{N_2}{(N_1^2 + M_1^2)^{3/2}} \quad \kappa_3 = \frac{N_3^2 + M_3^2}{(N_1^2 + M_1^2)^5} \quad \kappa_4 = \frac{N_4 - M_4}{(N_1^2 + M_1^2)^{7/2}}$$